

## REVIEWS

**Applied Analysis of the Navier–Stokes Equations.** By C. DOERING and J. K. GIBBON. Cambridge University Press, 1995. 217 pp. ISBN 0-521-44568. £14.95 (paperback).

This interesting and valuable monograph presents the state of the art of the existence and uniqueness theory of the Navier–Stokes equations from a physics/dynamical systems point of view. Throughout, emphasis is laid on the light that this theory can shed on the still-grey area of turbulence. However, the book will raise the eyebrows of practical fluid dynamicists, because anyone who is to receive the wisdom contained must ignore the authors' disregard for traditional engineering and applied mathematics approaches. Quibbles with statements like 'viscosity is the tendency of a fluid to resist shearing motions' and 'the Euler equations are of a hyperbolic form' must not be allowed to interfere with the reader's concentration. If this forbearance can be shown, it will be seen that the authors have performed the following valuable tasks:

- (i) they have rendered the functional analysis, with all its gruesome spaces and norms, more accessible than has yet been possible in learned journals. Their sympathy for non-specialists even extends to providing physical interpretation of some of the more obscure norms in terms of concepts such as inertial and dissipative length scales;
- (ii) they have highlighted the way that some traditional statistical theories of turbulence are related to the functional analytic approach;
- (iii) they have shown how, for the rigorous theory, vortex stretching is indeed the stumbling block that has denied us the necessary bounds for the all-important velocity gradients in three-dimensional flow; even though boundary layers, shear layers and vortices receive little explicit mention, the concomitant *intermittency* emerges as the root of all evil for the mathematician.

All this is contained in chapters 5–9 of the book, culminating in the discussion of the holy grail of the fractal dimension of the universal attractor. The first four chapters, the most likely to engender *JFM*-reader rage, give some background to modelling fluid mechanics, a recapitulation of elementary ideas from dynamical systems and a very readable but one-sided twenty-page review of the statistical theory of turbulence. Indeed, this particular chapter and chapter 10 are singular in that they address practical questions; chapter 10 is the only one that considers boundaries that generate vorticity and contains estimates for the rate of dissipation of the energy thus created.

The central chapters 5–9 are motivated by the modern theory initiated by Leray. They are dominated by Galerkin representations to the extent that a newcomer might think all length scales in fluid flows are governed by the eigenvalues of Laplace's equation with periodic boundary conditions. Also the theory is intensely impractical because of the restriction to spatially periodic boundary conditions and, despite the authors' best efforts, there seems to be no way for the norms not to get as complicated in the eyes of an asymptoticist as triple-deck theory is to a physicist. But it cannot be denied that the modern approach offers a completely new slant to the traditionally physically defined difficulties in understanding turbulence: the way which wriggling around the massive obstacle of the velocity gradient estimates has led mathematicians to think more in terms of minimal assumptions and *a posteriori* verification than direct predictions of solvability is also revealing.

Overall, the authors have done a thoughtful job of expounding ideas that are not

close to the hearts of many applied mathematicians yet are capable of providing new basic and computational understandings. There is one nasty comment that to many readers the distinction between weak and strong solutions may seem 'little more than a mathematical formality of no real consequence or practical importance'. But for most of the time, the thought-provoking style and accessibility will surely encourage researchers working on other irregularly behaved models in applied mathematics to look at their work in the light of modern partial differential equations theory and, as such, the book will occupy a niche in the annals of fluid mechanics.

J. R. OCKENDON

**Waves and Distributions.** By T. JONSSON and J. YNGVASON. World Scientific, 1995. 186 pp. ISBN 98102097461. £33.

The book is based on a set of notes for a course given to undergraduates in physical science and engineering with the aim of describing mathematical methods suitable for tackling physical problems. The chosen format means that a lot of topics can be mentioned in the 186 pages of the book, but it also means that much of the treatment is descriptive and explanatory. Proofs that are lengthy or require sophisticated techniques are excluded and readers are referred to other texts to find detailed confirmation of the numerous statements made without proof. While this method allows a quick introduction to the phenomena it does leave the reader who likes to see an argument substantiated feeling somewhat unanchored. The student who wishes to draw full benefit from this book will either have a good familiarity with a fair amount of mathematics or be prepared to undertake extensive additional reading.

The book assumes that the reader is already well-versed in electromagnetic theory (the discussion of electromagnetism starts with Maxwell's equations) but to be unacquainted with continuum mechanics. Therefore, the first chapter provides an introduction to fluid mechanics and elasticity. Ideas come in at a pretty rapid rate (sound waves are reached in six pages) and then the pace slows for a discussion of stress and strain. The approach to the stress tensor is not a direct physical one from the forces acting on a surface element but a mathematical one via a linear mapping of the normal vector. Likewise the strain tensor is reached by manipulation of linear mappings. The final section obtains the equations of dilational and shear waves in a homogeneous isotropic elastic medium. The chapter concludes, as does every chapter, with a set of testing exercises.

Chapter 2 gives a brief derivation from Maxwell's equations (in Gaussian units rather than mks) of the equations for the scalar and vector potentials. There is no mention of the Hertz vector.

In Chapter 3 the topic is plane waves. After the customary one-dimensional solution and harmonic version there is a short account of polarization. Boundary conditions are then introduced leading to the usual reflection and transmission coefficients for a plane interface irradiated in one case by a plane sound wave and in the other by a plane dilatational wave.

There are no waves in Chapter 4, which brings in distributions. The treatment is *ab initio* based on functionals as in Schwartz's book, but all difficult proofs are consigned to references elsewhere. For the reader who wants a quick introduction this is a very readable account of the subject. The principal target of the chapter is to get to the Fourier transform of a tempered distribution since this is the main tool employed in later chapters.

Waves in unbounded homogeneous media are tackled in Chapter 5 by distributions

and Fourier transforms. The different behaviour in space of odd and even dimensions is brought out clearly. The initial value problem and uniqueness are treated in some detail. Solutions in spherical polars and associated multipoles are described fully. The only boundary-value problems considered are those of cavities and waveguides (some engineers might not recognize the book's definition: if  $S = R \times R$ ,  $R \subset R^{d-1}$  then  $S$  is a waveguide with cross-section  $R$ ). The modal analysis is kept to a general level and is solely for Helmholtz's equation. No actual modes in a specific cavity or waveguide are determined.

The last chapter is a neat treatment of dispersion, emphasizing the difference between phase, group and signal velocities. It finishes with the properties of surface waves on a liquid interface.

To sum up, the authors have written clearly about the material they have selected and the knowledgeable should find it straightforward reading. It could serve as a valuable notebook for a student who has already taken a course on the subject matter but it would be tough going without supplementary instruction.

D. S. JONES

**Hot-Wire Anemometry.** By H. H. BRUUN. Oxford University Press, 1995. 507 pp. ISBN 0 19 856 342 6. £60.

A book on hot-wire anemometry, a single-point measuring technique which many scientists think to be outdated in our age of PIV and other field methods, appears on the scene in 1995. Is this timely or even necessary? Yes, it is. After the passing away or retirement of most of the fathers of hot-wire techniques, and in view of the waning ability of many of the younger generation to read, someone had to collect the labour and some of the experience of that past generation. This has been done in an excellent manner by H. H. Bruun in his new book on hot-wire anemometry: comprehensive, clear and easy of access for students, engineers, and researchers engaged in the practical use of any aspect of thermal anemometry.

Several good and useful books and extensive review papers on this topic have been published in the past but this book sets a standard which will be difficult to surpass. One of the reasons for this lies in its goal to be 'of direct value to people with no or very little practical knowledge of HWA as well as to researchers with considerable experience in HWA'. One example for the benefit of the former group of users is chapter 3 'which introduces the logical framework for a computer-based HWA system and identifies the individual steps in the complete experimental procedure ranging from probe selection through to presentation of analysed data'. The other chapters deal with basic principles of HWA: one-, two-, and three-component velocity measurements; temperature effects; HWA techniques for reversing flow and for the near-wall region; two-phase flows, gas mixtures, and compressible flows; velocity measurements; conditional sampling techniques; time-series analysis. The book concludes with an impressive list of references filling pages 446–501 with 1993 as the latest year.

The test of the pudding is in the eating, i.e. in using the book in the laboratory, but if the reviewer takes the judgement of his research students as a prediction of the success of this book, then the odds are excellent. One even quoted G. C. Lichtenberg: 'Wer zwei Paar Hosen hat, mache eine zu Geld und schaffe sich dieses Buch an'.

H. H. FERNHOLZ